Details on the Sample Size Calculator for the Cochran-Armitage Test

The Cochran-Armitage trend test in proportions involves an ordered set of groups for which we test for a linear trend in the proportions responding as we go across the ordering. The linear trend in the probabilities of response is given by the expression:

$$p_i = a + bd_i ,$$

where we test for a non-zero slope relating the probabilities to the numeric assigned to each of the ordered groups. For details on the test statistic see Agresti (2002) and see similar details as well as SAS implementation of the test at this link:

http://www.lexjansen.com/pharmasug/2007/sp/sp05.pdf

Nam (1987) notes that the test statistic is equivalent to using a linear logistic model instead and testing for the slope associated with ordered levels in this logistic model. Nam evaluates the sample size using this logistic model.

Using the same notation as in Nam, we define p_i as the probability of response, q_i as the probability of non-response, d_i as the ordered numeric assigned to a group (typically the actual dose when looking at dose levels or ordinally ordered as equal to the subscript *i*) and r_i as the multiple of the sample size in a group n_i to that in the control n_0 for $i = 0$ to $(k-1)$ (*k* groups including control). Further $\bar{d}$ is the average of the d_i's, $z_{1-\alpha}$ is the $(1-\alpha)^{th}$ quantile the standard normal distribution, $z_{1-\beta}$ is the $(1-\beta)^{th}$ quantile of the standard normal distribution, Δ is the difference between successive d_i's and $p = \left(\sum r_i p_i\right) / \left(\sum r_i\right)$. Nam (1987) provide the following expression for the continuity corrected sample size for the Cochran-Armitage Trend test in the control arm

$$n_0 = (n_0^* / 4)\left[1 + \sqrt{1 + 2\Delta / (A n_0^*)}\right]^2 ,$$

where n_0^* is the sample size in the control arm without the continuity correction and is give by

$$n_0^* = \left[z_{1-\alpha}\sqrt{pq\left[\sum r_i (d_i - \bar{d})^2\right]} + z_{1-\beta}\sqrt{\sum r_i p_i q_i (d_i - \bar{d})^2}\right]^2 / A^2 ,$$

for $A = \sum r_i p_i (d_i - \bar{d})$.

References:

1) Nam Jun-mo (1987), *A simple approximation for calculating sample sizes for detecting linear trend in proportions*. Biometrics. Vol 43, No 3, pp701-705.
2) Agresti Alan (2002), *Categorical Data Analysis*. John Wiley and Sons, Hoboken, NJ. Pp181-182.

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